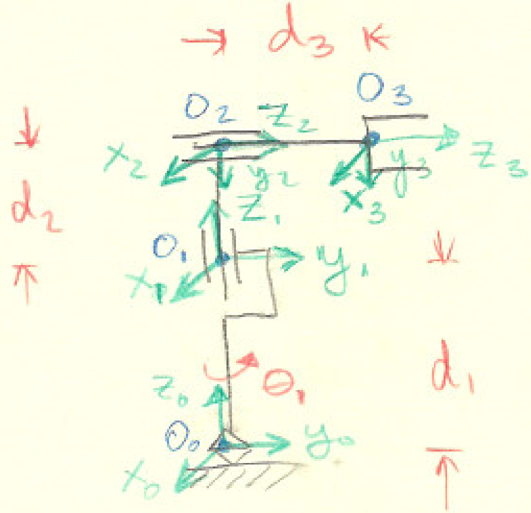
EX: 3 link cylindrical robot.



$x_0 \dots x_3$ are orthogonal to board.

What we are after is ${}^0_3H$

$${}^0_3H = \left[\begin{array}{c|c} {}^0_3R & {}^0_3d \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$= A_1 A_2 A_3$$

$$= {}^0_1H \, {}^1_2H \, {}^2_3H$$

Link	a_i	d_i	α_i	θ_i
1	0	d_1	0	θ_1^*
2	0	d_2^*	$-90°$	0
3	0	d_3^*	0	0

$$A1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_3H = \begin{bmatrix} C_{\theta_1} & 0 & -S\theta_1 & -S_{\theta_1}d_3 \\ S_{\theta_1} & 0 & C\theta_1 & C_{\theta_1}d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 6 & 0 & 0 & 1 \end{bmatrix}$$

Note $a \cdot p = a^T p$

dot product.

If $R \in \underbrace{SO(3)}$ set of orthogonal matrices of order 3.

If $a, b \in \mathbb{R}^3$

$$R(a \times b) = Ra \times Rb$$

Ex let us consider 2 vectors

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

x & y are expressed in a frame

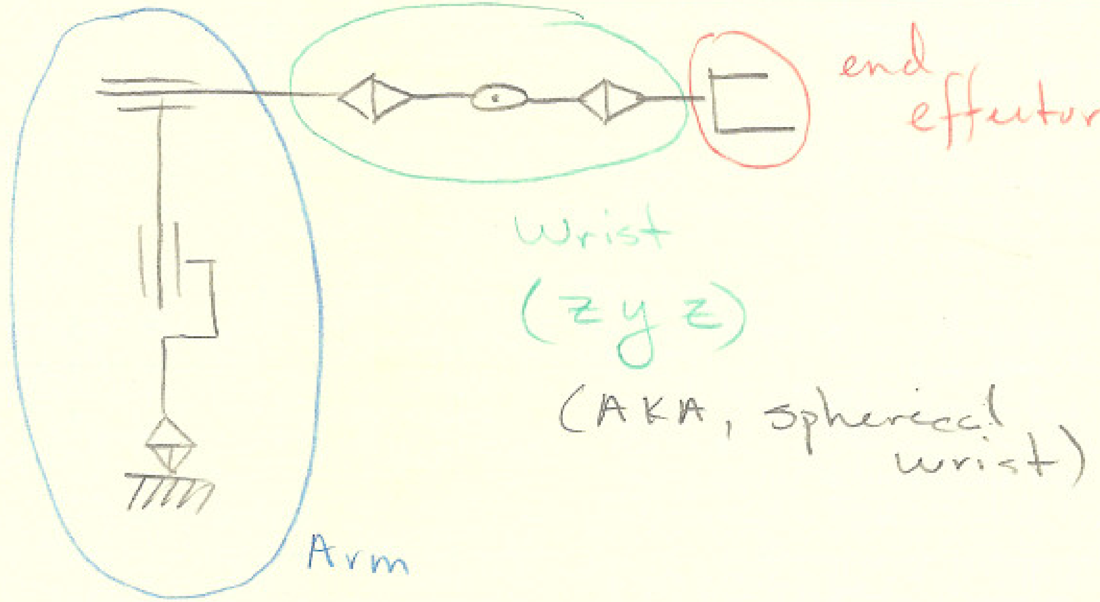
$(o, \hat{i}, \hat{j}, \hat{k})$

$$C = x y z = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$$

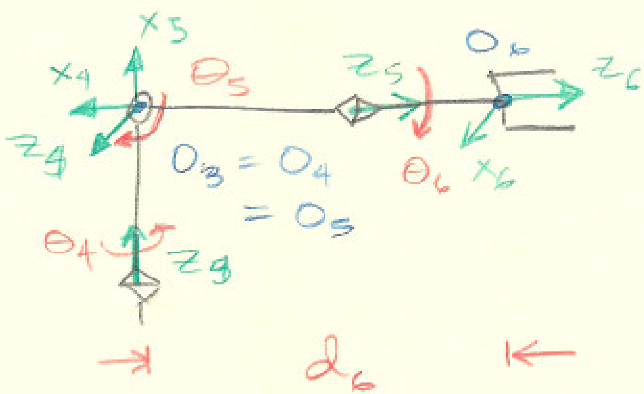
$$= (x_2 y_3 - x_3 y_2)\hat{i} + (x_3 y_1 - x_1 y_3)\hat{j}$$

$$+ (x_1 y_2 - x_2 y_1)\hat{k}$$

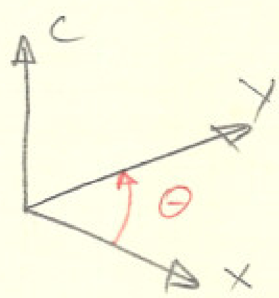
$$= \begin{bmatrix} x_2 y_3 - x_3 y_2 \\ x_3 y_1 - x_1 y_3 \\ x_1 y_2 - x_2 y_1 \end{bmatrix}$$



Spherical wrist



Link	a_i	d_i	α_i	θ_i
4	0	0	$-90°$	θ_4^*
5	0	0	$90°$	θ_5^*
6	0	d_6	0	θ_6^*



$$x \times y = c$$

$$\|c\| = \|x\| \, \|y\| \sin\theta$$

For any $R \in SO(3)$ $a \in \mathbb{R}^3$

$$\boxed{R\,S(a)\,R^T = S(Ra)}$$

important property

Suppose that a rotation matrix R is a function of a single variable θ

$$R(\theta) \in SO(3) \quad \text{for any } \theta$$

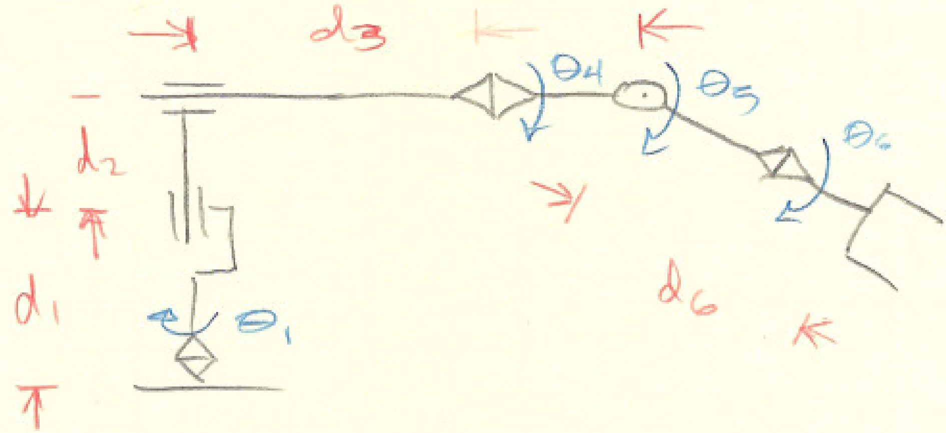
Since R is orthagonal then

$$R^T(\theta)\,R(\theta) = I \qquad (1)$$

differentiating (1) we get

$$\frac{dR(\theta)}{d\theta} R^T(\theta) + R(\theta)\frac{dR^T(\theta)}{d\theta} = 0 \qquad (2)$$

EX: Cylindrical arm w spherical wrist.



There are 6 degrees of freedom (DOF)

$${}^0_6H = {}^0_3H \; {}^3_6H$$

$$= A_1 \cdots A_6$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = C_1C_4C_5C_6 - C_1S_4S_6 + S_1S_5C_6$$

$$r_{21} = S_1C_4C_5C_6 - S_1S_4S_6 - C_1S_5C_6$$

$$r_{31} = \cdots$$